

# R\*\*2 correction to BMPV black hole entropy from Kerr/CFT correspondence

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ABSTRACT: Following Kerr/CFT correspondence, we compute the entropy of five-dimensional supersymmetric rotating BMPV black holes. We successfully reproduce Iyer-Wald formula in the presence of Gauss-Bonnet terms from the viewpoint of microscopic CFT. This further supports the higher-derivative version of Kerr/CFT prescription proposed in arXiv:0903.4176 for four-dimensional extremal Kerr black holes.

## 1. Introduction

A microscopic entropy counting programme called Kerr/CFT correspondence was proposed by Guica et al. in [1]. They have examined 4D extremal Kerr black holes whose near-horizon geometry has  $SL(2, R) \times U(1)$  isometry. Further extension can be found in [2]-[21]. Remarkably, the tree-level black hole entropy  $S_0$  predicted by Bekenstein-Hawking area law was reproduced by applying Cardy's formula to the dual 2D CFT:

$$S_0 = \frac{\pi^2}{3} c T_{FT} \quad (1.1)$$

where  $T_{FT}$  denotes Frolov-Thorne temperature [22].

Let us momentarily clarify the nature of Kerr/CFT correspondence. Compared with the BTZ/CFT case [23] pioneered earlier by Brown and Henneaux,<sup>1</sup> here one takes into account the asymptotic symmetry group (ASG) of the near-horizon black hole geometry instead of its asymptotically-far one. We will adopt the terminology *asymptotic* Killing vector (instead of Killing vector) especially for the former case under consideration. In addition, the central charge  $c$  in [23] did arise from the enhanced  $SL(2, R)_L \times SL(2, R)_R$ , the isometry of the asymptotic 3D BTZ (or  $AdS_3$ ), to two copies of (chiral and anti-chiral) Virasoro algebras. In contrast to this,  $c$  in (3.2) has a rather different nature. That is, given the above asymptotic Killing vector field like (3.1) infinitely many Fourier modes  $\zeta_n$  are identified with generators  $L_n$  of Virasoro algebra whose central charge (3.2) is determined completely by the near-horizon metric and (3.1).

In this Letter, within the framework of Kerr/CFT correspondence we focus on the entropy of 5D BMPV black holes in the presence of  $R^2$ -curvature corrections.<sup>2</sup> In [14, 15], this direction has been explored in the context of 4D extremal Kerr ones. We still rely on Cardy's formula (1.1) but modify the central charge  $c$  due to the inclusion of Gauss-Bonnet terms. It is in [28, 29, 30] that the explicit expression of  $c$  in the presence of higher-derivative corrections has been spelt out. Combined with Frolov-Thorne temperature analyzed in [8, 7] for BMPV black holes, we found that (1.1) coincides perfectly with Iyer-Wald formula [31, 32, 33].

We organize this Letter as follows. In Section 2, we describe some basic aspects of BMPV black holes. Iyer-Wald entropy formula in the presence of Gauss-Bonnet terms is also reviewed. In Section 3, we evaluate BMPV entropy microscopically via Kerr/CFT correspondence as stated above. We end up this Letter with a summary in Section 4.

## 2. The BMPV black hole entropy

The BMPV black hole was first constructed in [34]. As shown by Kallosh et al. [35], it can get embedded in 5D  $\mathcal{N} = 2$  supergravity coupled to one vector multiplet, and preserves

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<sup>1</sup>See Saida and Soda [24] for the extension of Brown-Henneaux to the inclusion of Gauss-Bonnet terms.

<sup>2</sup>See also [25, 26, 27] for the derivation of subleading corrections from other viewpoints.

one-half supersymmetry. The metric of the BMPV black hole is

$$ds^2 = - \left(1 - \frac{\mu}{r^2}\right)^2 dt^2 + \frac{dr^2}{\left(1 - \frac{\mu}{r^2}\right)^2} - \frac{\mu a}{r^2} \left(1 - \frac{\mu}{r^2}\right) \sigma_3 dt - \frac{\mu^2 a^2}{4r^4} \sigma_3^2 + \frac{r^2}{4} d\Omega_3^2 \quad (2.1)$$

where

$$\begin{aligned} \sigma_3 &= d\varphi + \cos\theta d\psi, & d\Omega_3^2 &= d\theta^2 + \sin^2\theta d\psi^2 + \sigma_3^2, \\ 0 \leq \theta &< \pi, & 0 \leq \psi &< 2\pi, & 0 \leq \varphi &< 4\pi. \end{aligned}$$

Here,  $\frac{1}{4}d\Omega_3^2$  is the line element of a unit  $S^3$ .

The lowest scalar component of the vector multiplet can be set to some constant. Because of this fact, the gaugino equation implies a vanishing gauge field in the vector multiplet. On the other hand, the graviphoton gauge potential in the graviton multiplet is

$$A = B(r)dt + C(r)\sigma_3, \quad (2.2)$$

where

$$B(r) = \frac{\sqrt{3}\mu}{2r^2}, \quad C(r) = -\frac{\sqrt{3}\mu a}{4r^2}.$$

The conserved charges of the BMPV black hole (2.1) can be measured at the asymptotic spatial infinity according to [36]. Thus, the conserved angular momentum along  $\varphi$  and the electric charge measured at asymptotic infinity of the BMPV black hole (2.1) are

$$J \equiv J_\varphi = -\frac{1}{16\pi G_5} \int_\infty * \nabla \xi^\varphi = -\frac{\pi a \mu}{4G_5}, \quad (2.3)$$

$$Q = \frac{1}{4\pi G_5} \int_\infty * dA = -\frac{\sqrt{3}\pi\mu}{2G_5} \quad (2.4)$$

where  $\nabla_\mu(\xi^\varphi)_\nu dx^\mu \wedge dx^\nu$  is abbreviated to  $\nabla \xi^\varphi$  with  $\xi^\varphi$  being a Killing vector field  $\frac{\partial}{\partial \varphi}$ . Here,  $x^\mu$  represents the coordinates  $(t, r, \theta, \psi, \varphi)$ . We have also expressed the 5D Newton constant as  $G_5$ . The ADM mass of the BMPV black hole,  $M_{ADM} = \frac{3\pi\mu}{4G_5}$ , is proportional to  $Q$  as required by supersymmetry. One can also see that there is no angular momentum along  $\psi$ .

Another realization of the BMPV black hole is through compactifying the ten-dimensional Type IIA string theory on a Calabi-Yau threefold  $X$  [37]. Wrapped on  $X$  is a D0-D2-D4-D6 brane system with  $(q_0, q_A, p^A, p^0)$  with  $A = 1, \dots, h^{1,1}(X)$  indicating their RR charges (or the numbers). In the case of the BMPV black holes, one has  $(q_0, q_A, p^A, p^0) = (q_0, q_A, 0, 1)$ . Due to one single D6-brane, the eleven-dimensional M-theory lift of this brane system amounts to placing the resultant 5D charged and rotational black hole at the center of a Taub-NUT space  $TN_4$  (in order to preserve supersymmetry). Notify that  $q_0 \propto J_\varphi$  is the spin over the  $S^1$  bundle of  $TN_4$ . This stringy construction makes natural both the appearance of  $S^3 \subset TN_4$  in (2.1) and the statement of no angular momentum along  $\psi$ . In addition,  $q_A$  is related to  $Q$  by  $q_A = 3Q D_{ABC} Y^B Y^C$  where  $D_{ABC}$  is the triple intersection

number of  $X$  and  $Y^A$  stands for horizon values of scalar components in  $\mathcal{N} = 2$  vector multiplets. They are normalized by  $D_{ABC}Y^AY^BY^C = 1$ .

In order to consider the near-horizon limit, let  $\tilde{r} = r - \sqrt{\mu}$ . In the near-horizon limit  $\tilde{r} \rightarrow 0$ , the BMPV black hole metric (2.1) becomes

$$ds^2 = - \left( \frac{2\tilde{r}}{\sqrt{\mu}} \right)^2 dt^2 + \left( \frac{\sqrt{\mu}}{2\tilde{r}} \right)^2 d\tilde{r}^2 - \frac{2a}{\sqrt{\mu}} \tilde{r} \sigma_3 dt + \frac{\mu - a^2}{4} \sigma_3^2 + \frac{\mu}{4} (d\theta^2 + \sin^2 \theta d\psi^2). \quad (2.5)$$

Obviously, when  $J = 0$  ( $a = 0$ ) (2.5) reduces to simply a direct product:  $AdS_2 \times S^3$ . For generic  $J \neq 0$  cases, the bosonic isometry group of (2.5) gets broken down to  $SU(1,1) \times SU(2) \times U(1)$  [38]. By completing the square term  $-(\frac{2\tilde{r}}{\sqrt{\mu}}dt + \frac{a}{2}\sigma_3)^2$ , the near-horizon topology of the BMPV black hole becomes  $AdS_2$  fibered over  $S^3$  [39]. This may be a sign of the existence of certain putative dual 2D chiral CFT. From (2.5) it is easily seen that the tree-level entropy is

$$S_0 = \frac{\text{Area}}{4G_5} = \frac{\pi^2}{2G_5} \mu \sqrt{\mu - a^2}. \quad (2.6)$$

## 2.1 Entropy from the Iyer-Wald formula

Let us see how  $R^2$ -curvature corrections modify (2.6). When it comes to the Gauss-Bonnet terms, the result has actually been calculated in [40]. Our goal here is to reproduce the known result and meantime set up some notational conventions to be used in the next section.

The 5D  $\mathcal{N} = 2$  supergravity can arise from compactifying 11D M-theory on Calabi-Yau threefolds  $X$  where the number of vector multiplets is  $h^{1,1}(X)$ . The resultant Einstein-frame low-energy effective action may necessarily contain higher-derivative corrections [41, 42] such as the familiar Gauss-Bonnet term:

$$\delta\mathcal{L} = \xi(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2) \quad (2.7)$$

where  $\xi$  is proportional to  $c_{2A}Y^A$ . As mentioned before,  $Y^A$  denotes the scalar component in vector multiplets evaluated at the horizon.  $c_{2A}$  is given by

$$c_{2A} = \int_X c_2(X) \wedge \omega_A \quad (2.8)$$

where  $c_2(X)$  is the second Chern class of  $X$  and  $\omega_A \in H^{1,1}(X)$ . What appeared in (2.1)-(2.2) correspond to the situation where one considers a special family of Calabi-Yau threefolds subject to  $h^{1,1}(X) = 1$ . We will concentrate only on this case.

We call  $S_1$  the correction to the tree-level entropy  $S_0$  due to the higher-derivative corrections  $\delta\mathcal{L}$ . The correction  $S_1$  to the tree-level entropy  $S_0$  is computed by means of the Iyer-Wald formula<sup>3</sup> [31, 32, 33]:

$$S_1 = -2\pi \int_{\Sigma} Z_{\mu\nu\rho\sigma} \epsilon^{\mu\nu} \epsilon^{\rho\sigma} \text{vol}(\Sigma), \quad Z_{\mu\nu\rho\sigma} = \frac{\partial \delta\mathcal{L}}{\partial R^{\mu\nu\rho\sigma}} \quad (2.9)$$

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<sup>3</sup>Certainly, the tree-level entropy formula (2.6) can also be recovered by considering the Einstein-Hilbert term for the Lagrangian density, namely  $Z_{\mu\nu\rho\sigma} = \partial(\sqrt{g}R)/\partial R^{\mu\nu\rho\sigma}$ .

where  $\text{vol}(\Sigma)$  stands for the volume form over the horizon  $\Sigma$ . Here,  $\epsilon^{\mu\nu}$  ( $\epsilon^{\mu\nu}\epsilon_{\mu\nu} = -2$ ) denotes the binormal to  $\Sigma$ . Plugging (2.7) into (2.9), we have

$$Z_{\mu\nu\rho\sigma} = \xi(2R_{\mu\nu\rho\sigma} - 2(R_{\mu\rho}g_{\nu\sigma} - R_{\mu\sigma}g_{\nu\rho} + R_{\nu\sigma}g_{\mu\rho} - R_{\nu\rho}g_{\mu\sigma}) + R(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho})). \quad (2.10)$$

Note that the form of (2.10) arises according to the guideline that indices of  $Z_{\mu\nu\rho\sigma}$  should obey

$$Z_{\mu\nu\rho\sigma} = -Z_{\nu\mu\rho\sigma} = -Z_{\mu\nu\sigma\rho} = Z_{\nu\mu\sigma\rho} = Z_{\rho\sigma\mu\nu}, \quad (2.11)$$

as those of the Riemann tensor  $R_{\mu\nu\rho\sigma}$  do.

In order to carry out the computation in (2.9) with respect to the tree-level metric (2.5), it may be easier to take an orthonormal basis instead of  $dx^\mu$ . An orthonormal basis  $e^{\hat{a}} = e^{\hat{a}}{}_\mu dx^\mu$ , ( $\hat{a} = \hat{t}, \hat{r}, \hat{\theta}, \hat{\psi}, \hat{\varphi}$ ) of (2.5) is

$$\begin{aligned} e^{\hat{t}} &= \frac{2\tilde{r}}{\sqrt{\mu - a^2}} dt, & e^{\hat{r}} &= \frac{\sqrt{\mu}}{2\tilde{r}} d\tilde{r}, & e^{\hat{\theta}} &= \frac{\sqrt{\mu}}{2} d\theta, \\ e^{\hat{\psi}} &= \frac{\sqrt{\mu}}{2} \sin\theta d\psi, & e^{\hat{\varphi}} &= \frac{\sqrt{\mu - a^2}}{2} \left( d\varphi + \cos\theta d\psi - \frac{4\tilde{r}a}{\sqrt{\mu}(\mu - a^2)} dt \right). \end{aligned} \quad (2.12)$$

If one uses the orthonormal basis (2.12), the non-zero components for the binormal  $\epsilon^{\hat{a}\hat{b}}$  simply become  $\epsilon^{\hat{t}\hat{r}} = -\epsilon^{\hat{r}\hat{t}} = 1$  and the others are zero. Then, the Iyer-Wald formula can be simply written by

$$S_1 = -2\pi \int_{\Sigma} (4Z_{\hat{t}\hat{r}\hat{t}\hat{r}}) e^{\hat{\theta}} \wedge e^{\hat{\psi}} \wedge e^{\hat{\varphi}}. \quad (2.13)$$

When one explicitly inserts the near-horizon metric (2.5) to (2.13), one obtains the correction  $S_1$  to the tree-level entropy  $S_0$  (2.6),

$$S_1 = 32\pi^3 \xi \mu \sqrt{\mu - a^2} \frac{(3\mu + a^2)}{\mu^2}. \quad (2.14)$$

### 3. Entropy from the Kerr/CFT correspondence

Having obtained (2.14) from the geometric viewpoint, we go to provide a statistical (or microscopic) derivation by means of the Kerr/CFT prescription.

#### 3.1 Central charge of dual 2D CFT

As shown in [1], because a suitable asymptotic boundary condition is imposed on the near-horizon metric there appears a series of vector fields which preserves them. We may call them *asymptotic* Killing vector fields. One has to tell the difference between them and usual Killing vector fields as advocated in Section 1. The asymptotic boundary condition for the near-horizon metric (2.5) has been worked out in [7]. It is preserved by the following vector field:

$$\zeta_n = \sum_{\alpha} \zeta_n^{\alpha} \partial_{\alpha} = -e^{-in\varphi} (\partial_{\varphi} + in\tilde{r}\partial_{\tilde{r}}). \quad (3.1)$$

$\zeta$ 's satisfy the Witt algebra  $i[\zeta_m, \zeta_n] = (m - n)\zeta_{m+n}$ .

To yield the Virasoro algebra one has to add a central term. The central extension can be accomplished by using an asymptotic charge associated to the asymptotic Killing vector field (3.1). The general formula for the asymptotic charge with respect to a given Lagrangian density including higher-derivative terms has been obtained in [28, 29, 30]:

$$c = 12i \left\{ -2 \int_{\Sigma} \left[ \mathbf{X}_{\alpha\beta} \mathcal{L}_{\zeta_n} \nabla^{\alpha} \zeta_{-n}^{\beta} + (\mathcal{L}_{\zeta_n} \mathbf{X})_{\alpha\beta} \nabla^{[\alpha} \zeta_{-n}^{\beta]} + \mathcal{L}_{\zeta_n} \mathbf{W}_{\alpha} \zeta_{-n}^{\alpha} \right] - \int_{\Sigma} \mathbf{E}[\mathcal{L}_{\zeta_n} \phi, \mathcal{L}_{\zeta_{-n}} \phi; \bar{\phi}] \right\} \Big|_{n^3} \quad (3.2)$$

where by  $|_{n^3}$  we mean that only terms proportional to  $n^3$  will be extracted, and  $\mathcal{L}_{\zeta_n}$  denotes the Lie derivative with respect to  $\zeta_n$  (3.1).  $\mathbf{X}_{\alpha\beta}$  and  $\mathbf{W}^{\alpha}$  are defined by

$$\begin{aligned} (\mathbf{X}_{\alpha\beta})_{\lambda\mu\nu} &= -\epsilon_{\rho\sigma\lambda\mu\nu} Z^{\rho\sigma}_{\alpha\beta}, \\ (\mathbf{W}^{\alpha})_{\lambda\mu\nu} &= 2 \left( \nabla_{\beta} \mathbf{X}^{\alpha\beta} \right)_{\lambda\mu\nu}. \end{aligned} \quad (3.3)$$

Furthermore, the explicit form of the  $\mathbf{E}$ -term in (3.2) was obtained in [15]; namely,

$$\mathbf{E} \equiv \mathbf{E}_{\lambda\mu\nu} = \epsilon_{\rho\sigma\lambda\mu\nu} \frac{1}{2} \left( -\frac{3}{2} Z^{\rho\sigma\gamma\eta} \delta g_{\gamma}{}^{\kappa} \wedge \delta g_{\kappa\eta} + 2 Z^{\rho\gamma\eta\kappa} \delta g_{\gamma\eta} \wedge \delta g^{\sigma}{}_{\kappa} \right), \quad (3.4)$$

where  $\phi$  is any field including  $g_{\mu\nu}$ . Furthermore,  $\mathbf{E}[\mathcal{L}_{\zeta_n} \phi, \mathcal{L}_{\zeta_{-n}} \phi; \phi]$  is defined as

$$\mathbf{E}[\mathcal{L}_{\zeta_n} \phi, \mathcal{L}_{\zeta_{-n}} \phi; \phi] \equiv (\mathcal{L}_{\zeta_{-n}} \phi \frac{\partial}{\partial \phi} + \partial_{\mu} \mathcal{L}_{\zeta_{-n}} \phi \frac{\partial}{\partial (\partial_{\mu} \phi)} + \cdots)_{\perp} (\mathcal{L}_{\zeta_n} \phi \frac{\partial}{\partial \phi} + \partial_{\mu} \mathcal{L}_{\zeta_n} \phi \frac{\partial}{\partial (\partial_{\mu} \phi)} + \cdots)_{\perp} \mathbf{E} \quad (3.5)$$

where  $\perp$  denotes the interior product. From the direct evaluation of (3.5) by using (3.4), one explicitly has

$$\begin{aligned} \mathbf{E}[\mathcal{L}_{\zeta_n} \phi, \mathcal{L}_{\zeta_{-n}} \phi; \phi] &= \epsilon_{\rho\sigma\lambda\mu\nu} \frac{1}{2} \left( -\frac{3}{2} Z^{\rho\sigma\gamma\eta} \{ (\mathcal{L}_{\zeta_n} g)_{\gamma}{}^{\kappa} (\mathcal{L}_{\zeta_{-n}} g)_{\kappa\eta} - (\mathcal{L}_{\zeta_n} g)_{\kappa\eta} (\mathcal{L}_{\zeta_{-n}} g)_{\gamma}{}^{\kappa} \} \right. \\ &\quad \left. + Z^{\rho\gamma\eta\kappa} \{ (\mathcal{L}_{\zeta_n} g)_{\gamma\eta} (\mathcal{L}_{\zeta_{-n}} g)^{\sigma}{}_{\kappa} - (\mathcal{L}_{\zeta_n} g)^{\sigma}{}_{\kappa} (\mathcal{L}_{\zeta_{-n}} g)_{\gamma\eta} \} \right). \end{aligned} \quad (3.6)$$

With the expressions (3.3) and (3.6), one can explicitly evaluate (3.2).

Indeed, as shown in [15], by using the aforementioned definition of  $c$ , the Iyer-Wald formula gets completely reproduced at least in the 4D extremal Kerr cases. Here, nevertheless we clarify its validness even for the 5D BMPV black hole. As a matter of fact, it is highly non-trivial to check whether (3.2) still holds for the BMPV black hole because its near-horizon topology,  $S^1$ -fibration over  $AdS_2 \times S^2$ , definitely differs from that of the 4D extremal Kerr black holes.

### 3.2 Calculation

Again, let us take the orthonormal basis (2.12) during evaluating (3.2) for the sake of computational convenience. One can rewrite (3.1) by a basis  $e_{\hat{a}} = e_{\hat{a}}^{\mu} \partial_{\mu}$  where  $e_{\hat{a}}^{\mu} =$

$(e^{\hat{a}}_\mu)^{-1}$ . Then, the explicit form of  $\zeta_n$  in terms of the basis  $e_{\hat{a}}$  is

$$\zeta_n = -e^{-in\varphi} \left( \frac{\sqrt{\mu-a^2}}{2} e_{\hat{\varphi}} + in \frac{\sqrt{\mu}}{2} e_{\hat{r}} \right). \quad (3.7)$$

Equipped with the above *asymptotic* Killing vector (3.7), one manages to have

$$\left( \mathcal{L}_{\zeta_n} \nabla^{\hat{a}} \zeta_{-n}^{\hat{b}} \right) \Big|_{n^3} = \begin{pmatrix} 0 & \frac{ia}{2\sqrt{\mu-a^2}} & 0 & 0 & 0 \\ -\frac{ia}{2\sqrt{\mu-a^2}} & 0 & 0 & 0 & \frac{i(3a^2\mu-2\mu^2+(2a^2\mu-2\mu^2)\cot^2\theta)}{2\mu^{\frac{3}{2}}\sqrt{\mu-a^2}} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{i(3a^2\mu-2\mu^2+(2a^2\mu-2\mu^2)\cot^2\theta)}{2\mu^{\frac{3}{2}}\sqrt{\mu-a^2}} & 0 & 0 & 0 \end{pmatrix} \quad (3.8)$$

whose columns (rows) are labeled in order by  $\hat{t}, \hat{r}, \hat{\theta}, \hat{\psi}$  and  $\hat{\varphi}$ .

By combining both (3.8) and  $Z_{\hat{t}\hat{r}\hat{r}\hat{\varphi}} = 0$ , the first term in (3.2) becomes

$$\begin{aligned} c_{1st} &\equiv -24i \int_{\Sigma} \left[ 2Z_{\hat{t}\hat{r}\hat{t}\hat{r}} \left( \frac{ia}{2\sqrt{\mu-a^2}} \right) - 2Z_{\hat{t}\hat{r}\hat{r}\hat{t}} \left( -\frac{ia}{2\sqrt{\mu-a^2}} \right) \right] e^{\hat{\theta}} \wedge e^{\hat{\psi}} \wedge e^{\hat{\varphi}} \\ &= 48 \int_{\Sigma} \frac{a}{\sqrt{\mu-a^2}} Z_{\hat{t}\hat{r}\hat{t}\hat{r}} e^{\hat{\theta}} \wedge e^{\hat{\psi}} \wedge e^{\hat{\varphi}}. \end{aligned} \quad (3.9)$$

Equipped with Cardy's formula (1.1) and the Frolov-Thorne temperature [8, 7]

$$T_{FT} = -\frac{\sqrt{\mu-a^2}}{2\pi a}, \quad (3.10)$$

we have

$$\begin{aligned} S_1 &= \frac{\pi^2}{3} \int_{\Sigma} \left( 48Z_{\hat{t}\hat{r}\hat{t}\hat{r}} \frac{a}{\sqrt{\mu-a^2}} \right) \left( -\frac{\sqrt{\mu-a^2}}{2\pi a} \right) e^{\hat{\theta}} \wedge e^{\hat{\psi}} \wedge e^{\hat{\varphi}} \\ &= -8\pi \int_{\Sigma} Z_{\hat{t}\hat{r}\hat{t}\hat{r}} e^{\hat{\theta}} \wedge e^{\hat{\psi}} \wedge e^{\hat{\varphi}}. \end{aligned} \quad (3.11)$$

It precisely coincides with the Iyer-Wald result (2.13). Surely, we have assumed the validity of the Frolov-Thorne temperature even in the presence of the  $R^2$ -curvature corrections.<sup>4</sup> In view of (3.10), it seems that when  $a > 0$  the above  $T_{FT}$  of some *putative* 2D CFT become negative. This is a quite common issue encountered during applying Kerr/CFT correspondence. The sign dependence on  $a$  is merely an illusion because it has no real effect on a physical quantity like entropy  $S_1$  which is independent of the sign of  $a$ . Certainly, another choice of  $\zeta$  associated with  $\psi$  is possible but leads to a vanishing central charge. This situation which resembles the tree-level case encountered in [7] may be attributable to zero angular momentum along  $\psi$ .

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<sup>4</sup>It was argued in [15] that higher-derivative corrections have no effect on the Frolov-Thorne temperature of 4D extremal Kerr black holes.

Let us examine other contributions to the central charge  $c$  in (3.2). We will see that they cancel one another out eventually. Since we have already reproduced the Iyer-Wald result only from the first term of (3.2), the contributions from the other terms have to cancel one another out. In (3.2), the explicit evaluation can show that the second and third terms as a whole give

$$-24i \int_{\Sigma} \left[ (\mathcal{L}_{\zeta_n} \mathbf{X})_{\alpha\beta} \nabla^{[\alpha} \zeta_{-n}^{\beta]} + \mathcal{L}_{\zeta_n} \mathbf{W}_{\alpha} \zeta_{-n}^{\alpha} \right] \Big|_{n^3} = 384 \frac{a\sqrt{\mu-a^2}}{\mu^2} \int_{\Sigma} e^{\hat{\theta}} \wedge e^{\hat{\psi}} \wedge e^{\hat{\varphi}}. \quad (3.12)$$

On the other hand, one can explicitly compute (3.6) via the near-horizon metric (2.5), and the result is

$$-12i \int_{\Sigma} \mathbf{E}[\mathcal{L}_{\zeta_n} \phi, \mathcal{L}_{\zeta_{-n}} \phi; \bar{\phi}] \Big|_{n^3} = -384 \frac{a\sqrt{\mu-a^2}}{\mu^2} \int_{\Sigma} e^{\hat{\theta}} \wedge e^{\hat{\psi}} \wedge e^{\hat{\varphi}}. \quad (3.13)$$

Consequently, a perfect cancellation happens as expected. Namely, we have confirmed that the entropy obtained from the central charge (3.2) precisely reproduces the Iyer-Wald formula (2.9).

## 4. Summary

Our results are summarized as follows. Plugging into Cardy's formula of 2D CFT (1.1) the central charge spelt out in [28, 29, 30] and Frolov-Thorne temperature analyzed in [8, 7], we obtained the entropy of BMPV black holes when Gauss-Bonnet terms are present. This computation, though semiclassical, can be regarded as a microscopic derivation in contrast to Iyer-Wald formula.

Because there are two independent  $(\varphi, \psi)$ , it is  $\varphi$ -direction associated with non-zero angular momentum that gives us the finite central charge. This resembles much the tree-level (Einstein-Hilbert action) situation encountered in [7]. It will be interesting to understand this phenomenon further within a more general framework.

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## References

- [1] M. Guica, T. Hartman, W. Song and A. Strominger, “The Kerr/CFT Correspondence,” Phys. Rev. **D80** (2009) 124008, arXiv:0809.4266 [hep-th].



- [2] F. Loran and H. Soltanpanahi, “Near the horizon of 5D black rings,” *JHEP* **03** (2009) 035, arXiv:0810.2620 [hep-th].
- [3] H. Lu, J. Mei, and C. N. Pope, “Kerr/CFT Correspondence in Diverse Dimensions,” *JHEP* **04** (2009) 054, arXiv:0811.2225 [hep-th].
- [4] T. Azeyanagi, N. Ogawa, and S. Terashima, “Holographic Duals of Kaluza-Klein Black Holes,” *JHEP* **04** (2009) 061, arXiv:0811.4177 [hep-th].
- [5] T. Hartman, K. Murata, T. Nishioka, and A. Strominger, “CFT Duals for Extreme Black Holes,” *JHEP* **04** (2009) 019, arXiv:0811.4393 [hep-th].
- [6] D. D. K. Chow, M. Cvetič, H. Lu, and C. N. Pope, “Extremal Black Hole/CFT Correspondence in (Gauged) Supergravities,” *Phys. Rev.* **D79** (2009) 084018, arXiv:0812.2918 [hep-th].
- [7] H. Isono, T. -S. Tai, and W. -Y. Wen, “Kerr/CFT correspondence and five-dimensional BMPV black holes,” *Int. J. Mod. Phys.* **A24** (2009) 5659-5668, arXiv:0812.4440 [hep-th].
- [8] T. Azeyanagi, N. Ogawa and S. Terashima, “The Kerr/CFT Correspondence and String Theory,” *Phys. Rev. D* **79** (2009) 106009 [arXiv:0812.4883 [hep-th]].
- [9] J. -J. Peng and S. -Q. Wu, “Extremal Kerr black hole/CFT correspondence in the five dimensional Gödel universe,” *Phys. Lett.* **B673** (2009) 216-219, arXiv:0901.0311 [hep-th].
- [10] C. -M. Chen and J. E. Wang, “Holographic Duals of Black Holes in Five-dimensional Minimal Supergravity,” *Class. Quant. Grav.* **27** (2010) 075004 [arXiv:0901.0538 [hep-th]].
- [11] F. Loran and H. Soltanpanahi, “5D Extremal Rotating Black Holes and CFT duals,” *Class. Quant. Grav.* **26** (2009) 155019, arXiv:0901.1595 [hep-th].
- [12] G. Compere, K. Murata, and T. Nishioka, “Central Charges in Extreme Black Hole/CFT Correspondence,” *JHEP* **05** (2009) 077, arXiv:0902.1001 [hep-th].
- [13] D. Astefanesei and Y. K. Srivastava, “CFT Duals for Attractor Horizons,” *Nucl. Phys. B* **822** (2009) 283 [arXiv:0902.4033 [hep-th]].
- [14] C. Krishnan and S. Kuperstein, “A Comment on Kerr-CFT and Wald Entropy,” *Phys. Lett. B* **677** (2009) 326 [arXiv:0903.2169 [hep-th]].
- [15] T. Azeyanagi, G. Compere, N. Ogawa, Y. Tachikawa, and S. Terashima, “Higher-Derivative Corrections to the Asymptotic Virasoro Symmetry of 4d Extremal Black Holes,” *Prog. Theor. Phys.* **122** (2009) 355-384, arXiv:0903.4176 [hep-th].
- [16] M. Guica and A. Strominger, “Microscopic Realization of the Kerr/CFT Correspondence,” *JHEP* **1102** (2011) 010 [arXiv:1009.5039 [hep-th]].
- [17] G. Compere, W. Song and A. Virmani, “Microscopics of Extremal Kerr from Spinning M5 Branes,” *JHEP* **1110** (2011) 087 [arXiv:1010.0685 [hep-th]].
- [18] Y. Matsuo and T. Nishioka, “New Near Horizon Limit in Kerr/CFT,” *JHEP* **1012** (2010) 073 [arXiv:1010.4549 [hep-th]].
- [19] S. Carlip, “Extremal and nonextremal Kerr/CFT correspondences,” *JHEP* **1104** (2011) 076 [arXiv:1101.5136 [gr-qc]].
- [20] K. Goldstein and H. Soltanpanahi, “CFT Duals of Black Rings With Higher Derivative Terms,” arXiv:1108.4362 [hep-th].
- [21] M. Nakamura and N. Yokoi, “Near horizon superconformal symmetry of rotating BPS black holes in five dimensions,” arXiv:1109.6481 [hep-th].
- [22] V. P. Frolov and K. S. Thorne, “RENORMALIZED STRESS - ENERGY TENSOR NEAR THE HORIZON OF A SLOWLY EVOLVING, ROTATING BLACK HOLE,” *Phys. Rev.* **D39** (1989) 2125.
- [23] J. D. Brown and M. Henneaux, “Central Charges in the Canonical Realization of Asymptotic Symmetries: An Example from Three-Dimensional Gravity,” *Commun. Math. Phys.* **104** (1986) 207.

- [24] H. Saida and J. Soda, “Statistical entropy of BTZ black hole in higher curvature gravity,” *Phys. Lett. B* **471** (2000) 358 [gr-qc/9909061].
- [25] A. Castro and S. Murthy, “Corrections to the statistical entropy of five dimensional black holes,” *JHEP* **0906** (2009) 024 [arXiv:0807.0237 [hep-th]].
- [26] N. Banerjee, “Subleading Correction to Statistical Entropy for BMPV Black Hole,” *Phys. Rev. D* **79** (2009) 081501 [arXiv:0807.1314 [hep-th]].
- [27] A. Dabholkar, J. Gomes, S. Murthy and A. Sen, “Supersymmetric Index from Black Hole Entropy,” *JHEP* **1104** (2011) 034 [arXiv:1009.3226 [hep-th]].
- [28] G. Barnich and F. Brandt, “Covariant theory of asymptotic symmetries, conservation laws and central charges,” *Nucl. Phys. B* **633** (2002) 3-82, arXiv:hep-th/0111246.
- [29] G. Barnich and G. Compere, “Surface charge algebra in gauge theories and thermodynamic integrability,” *J. Math. Phys.* **49** (2008) 042901, arXiv:0708.2378 [gr-qc].
- [30] G. Compere, “Symmetries and conservation laws in Lagrangian gauge theories with applications to the mechanics of black holes and to gravity in three dimensions,” arXiv:0708.3153 [hep-th].
- [31] R. M. Wald, “Black Hole Entropy is the Noether Charge,” *Phys. Rev. D* **48** (1993) 3427, [arXiv:gr-qc/9307038].
- [32] V. Iyer and R. M. Wald, “Some Properties of Noether Charge and a Proposal for Dynamical Black Hole Entropy,” *Phys. Rev. D* **50** (1994) 846, [arXiv:gr-qc/9403028].
- [33] T. Jacobson, G. Kang and R. C. Myers, “On Black Hole Entropy,” *Phys. Rev. D* **49** (1994) 6587, [arXiv:gr-qc/9312023].
- [34] J. C. Breckenridge, R. C. Myers, A. W. Peet and C. Vafa, “D-branes and spinning black holes,” *Phys. Lett. B* **391** (1997) 93-98, [hep-th/9602065].
- [35] R. Kallosh, A. Rajaraman and W. K. Wong, “Supersymmetric rotating black holes and attractors,” *Phys. Rev. D* **55** (1997) 3246-3249, [hep-th/9611094].
- [36] R. C. Myers and M. J. Perry, “Black Holes in Higher Dimensional Space-Times,” *Annals of Physics* **172** (1986) 304.
- [37] D. Gaiotto, A. Strominger and X. Yin, “New connections between 4-D and 5-D black holes,” *JHEP* **0602** (2006) 024 [hep-th/0503217].
- [38] J. P. Gauntlett, R. C. Myers and P. K. Townsend, “Black holes of  $D = 5$  supergravity,” *Class. Quant. Grav.* **16** (1999) 1 [hep-th/9810204].
- [39] W. Li and A. Strominger, “Supersymmetric Probes in a Rotating 5D Attractor,” *Phys. Lett. B* **659** (2008) 407 [arXiv:hep-th/0605139].
- [40] M. Guica, L. Huang, W. Li and A. Strominger, “ $R^2$  corrections for 5-D black holes and rings,” *JHEP* **0610** (2006) 036 [hep-th/0505188].
- [41] S. Ferrara, R. R. Khuri and R. Minasian, “M theory on a Calabi-Yau manifold,” *Phys. Lett. B* **375** (1996) 81, [hep-th/9602102].
- [42] I. Antoniadis, S. Ferrara, R. Minasian and K. S. Narain, “ $R^4$  couplings in M and type II theories on Calabi-Yau spaces,” *Nucl. Phys. B* **507** (1997) 571, [hep-th/9707013].